Chapter 21

TIME VALUE OF MONEY AND QUANTITATIVE ANALYSIS

INTRODUCTION

The time value of money is an important concept for sound financial decisions. Present and future value computations provide quantitative techniques for determining the value of time in tax and financial decision-making and are essential in understanding:

1. the effect of time on the profitability of an investment;

2. how the projected value of an investment’s future economic returns affects the price that should be paid for it;

3. how to compute the value of an investment’s future economic return.

Sound financial decisions depend on an understanding of the basic mathematics of compound interest. This concept is essential in analyzing the financial consequences of almost any investment.

The concept of investment, by definition, implies a delay in consumption or enjoyment. For an individual to forego current consumption or enjoyment in favor of future consumption there must be some reward. That reward is called profit, which must be large enough to justify, at least in the mind of the investor, the expected delay. The measure of the profit is typically called the rate of return, or the rate of interest.

The concept of time value of money also is crucial to the larger picture of any cash flow or goal based financial planning. Planning for the future often involves projecting current and future cash flows and needs, such as for education funding, retirement planning, and estate planning. Effective planning for the future generally involves some estimation of present or future values, so as to put such outflows and inflows on a common footing that can be evaluated. In addition to rates of returns, rates of inflation, growth, tax, and probabilities may be modeled into such planning.

For more information on the technical aspects of the time value of money and of measuring investment returns, please consult our sister-publication the *Tools & Techniques of Investment Planning*.

HOW DOES IT WORK?

The particular type of investment that a given individual will make depends both on his financial resources and risk preferences. But regardless of preferences, there are three basic underlying rate of return principles that should govern every investment: (1) timing, (2) quality, and (3) quantity.

1. *Timing*. An early return of principal and income is preferable to a later return. For example, given two potential investments, one providing $1,000 now and the other providing $1,000 a year from now, the former investment should be the one selected.

This sooner is better than later concept will be referred to as timing. Tax law provides a good example of the advantages of timing. An important technique in income tax planning, and an integral part of a tax shelter investment, is the use of accelerated depreciation to recover the investor’s capital more quickly. The timing benefit of accelerated depreciation as compared to straight-line depreciation is the result of keeping tax dollars in the hands of the taxpayer for a longer period of time. This provides a quicker recovery of cost than would be possible through a slower form of depreciation.

2. *Quality*. An investment with less risk is preferable to one with greater risk. Therefore, if two alternative investments offer the same rate of return, but it is more likely that the principal of the second investment could be lost, the former investment should be the one selected. This likelihood of pay back concept is referred to as quality.

3. *Quantity*. Assuming investment risks are equal and the timing of the return is identical, the investment with the highest rate of return is preferable. Therefore, if two investments have equal timing and are equally risky, but one has a higher rate of return, it should be the one selected. This more-is-better-than-less concept will be referred to as quantity.

This chapter focuses on the principles inherent in the first and third of these three concepts (timing and quantity) and their interrelationship. It is only through a time adjusted analysis that an investment with a higher yield but a longer investment life can be effectively compared with an investment with a lower yield but a shorter life span.

In summary, the proper measure of the financial consequences of investment alternatives will focus not only on risk and the sum of the cash flows, but also reflect the differences in the expected timing of their receipt.

WHEN IS THE USE OF THIS TECHNIQUE INDICATED?

1. When an investor wishes to analyze an investment or compare alternative investments where any of the following factors is involved:

a) timing: When and/or how often must cash be put in and/or taken out of the investment?

b) quantity: At what interest rate is the investment earning or growing?

2. When planning for future financial needs (e.g., education, retirement, estate planning) while taking into consideration future income sources (e.g., investments, insurance, social security, retirement benefits).

ADVANTAGES

1. Makes it possible to determine whether an investor can afford to commit funds to a particular investment for the length of time required.

2. Permits a quantitative comparison of alternative investments with different rates of return and different investment life spans.

3. Allows future funding shortfalls to be identified and steps taken to address such needs as soon as possible.

DISADVANTAGES

1. Heavy reliance on quantitative techniques for evaluating alternatives may overshadow the need to review subjectively the appropriateness of an investment. There are times when an investor should rely on his gut feelings and play a hunch.

2. There are such a large number of measurement devices that the choice of the wrong one in a given situation may easily occur. For example, what would be the appropriate measuring tool to use in estimating the value of an investment 10 years from now? It would not be helpful to measure the future value of a series of payments during that period. It would, however, be appropriate to compute the future value (in 10 years) of a single, lump sum invested today.

3. The results of any quantitative analysis are only as good as the initial information provided. Financial analysis is subject to the same danger often raised by computer users: “garbage in, garbage out.” For example, in evaluating the present value of a retirement fund needed 10 years from now, it would be a mistake to use an interest rate of 14% if it is known that an investment can never earn at a rate greater than 10% per year. The use of a 14% interest assumption would provide an overly optimistic and misleading answer. In turn, this would lead to the possible underfunding of retirement needs.

PRESENT VALUE AND FUTURE VALUE

The right measurement device in a particular situation depends upon the nature of the problem to be solved. In this section, each of the basic time value of money concepts used in financial planning will be reviewed. These present value and future value concepts can also be used as the building blocks for other time value of money calculations.

I. Computing the Future Value of a Lump Sum

PROBLEM: If I have a lump sum of $\_\_\_\_\_\_ today, how do I calculate the value of that lump sum \_\_\_\_\_ years in the future assuming I earn \_\_\_% on my investment?

SOLUTION: Go to Appendix D, Future Value of a Lump Sum Table. This table reflects the amount $1 will be worth in a given number of years at various interest rates. For example, to find how much $10,000 invested today will be worth in 5 years, if it grows at the rate of 10% per year, you would multiply $10,000 by the factor found in the 5-year row under the 10% column. That factor is 1.6105. Therefore, $10,000 invested today will be worth $16,105 in 5 years.

This compound interest table provides a quick way to determine the future value of an investment or other asset such as a family home. However, this table contains only a limited number of years and interest rate assumptions. To compute the future value of an asset where the number of years or interest rate is not covered by the table, it may be necessary to utilize the following formula:

Equation FVLS



In this formula,

*n* = The Number of Years (Periods) in the Future

*FV* = Future Value Amount

*PV* = Present Value Amount

*i* = Interest Rate for Each Year (Period)

Applying the formula to our example,



The formula is a shortcut way of expressing the annual 10% compounding, in our example, of the initial $10,000 investment for the 5-year investment period. The actual, manual computation for the entire investment period can be charted as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Beginning Balance** |  | **Interest @ 10%** |  | **Ending Balance** |
| Year 1 | $10,000 | + | $1,000 | = | $11,000 |
| Year 2 | $11,000 | + | $1,100 | = | $12,100 |
| Year 3 | $12,100 | + | $1,210 | = | $13,310 |
| Year 4 | $13,310 | + | $1,331 | = | $14,641 |
| Year 5 | $14,641 | + | $1,464 | = | $16,105 |

The future value table in Appendix D carries compounding factors to four decimal places; other compound interest tables may show more or fewer decimal places and therefore cause the final result to be different. A further shortcoming of tables is evident when, for example, the interest rate or the compounding period is more complex, such as an interest rate of 10.375%, or a compounding period of 20 years, 7 months. In such cases, interpolation between numbers in the tables could be used.

A pocket calculator can handle the multiple computations necessary. Many financial calculators are sophisticated enough to perform the calculation by merely entering the ***P*** (principal), ***n*** (number of periods), and ***i*** (interest rate) numbers. In addition, personal computers easily can be programmed to perform the formula computations, as well as provide a printout of the future value at any interim period.

II. Computing the Present Value of a Future Lump Sum

PROBLEM: If I will have a lump sum of $\_\_\_\_\_\_ in \_\_\_ years, how do I calculate the present value of my investment, assuming it will earn interest at the rate of \_\_\_? In other words, what is the equivalent today of $\_\_\_\_\_\_ payable as a lump sum \_\_\_ years in the future?

SOLUTION: Go to Appendix A, Present Value of a Lump Sum Table. This table reflects the present value of $1 received at the end of a given number of years in the future at various interest rates. For example, to find out the present value of $100,000 to be received in 20 years, assuming a growth rate of 10% per year, multiply $100,000 by the factor found in the 20-year row under the 10% column. That factor is 0.1486. Therefore the $100,000 to be received in 20 years has a present value of $14,860.

This present value table was compiled using the following mathematical formula that can be used in cases that are not included in the table:

Equation PVLS



Again, in this formula, *PV* = Present Value Amount

*FV* = Future Value Amount

*n* = The Number of Years (Periods) Until the Future Payment

*i* = Interest Rate for Each Year (Period)

Applying the formula to our example,



A careful look at these first two formulas will reveal that they are really the same formula, but used to solve for different unknown elements. The first formula is used to determine the future value of a lump sum when the present value, number of years, and interest rate are known. The second formula is used to determine the present value when the future value, number of years, and interest rate are known. Either formula can be used to compute any one of the four factors in the equation, so long as the other three are known.

For example, either formula could be used if you wanted to know how many years it would take for a present value of $14,860 to grow to $100,000, if the annual interest rate earned on the amount was 10%. You could also use either the table in Appendix A or Appendix D, provided the three known elements can be located in the table. For example, using the table in Appendix A, the present value table, looking under the 10% column we can see that it will take 20 years for a present value of 0.1486 to grow to $1. Multiplying these amounts by 100,000, we see that it will take 20 years for a present value of $14,860 to grow to $100,000.

If the present value in the above example were $15,000, rather than $14,860, one could use the present value table only to estimate the actual period needed to compound the $15,000 to $100,000 at a 10% interest rate. In this case the formula would be needed to arrive at the actual compounding period required. It would then make sense to use a financial calculator or a computer to solve the problem.

Plugging this information into the second formula, the solution would look like this:





III. Calculating the Future Value of a Regular Series of Payments

PROBLEMS:

A. If, beginning today, I invest $\_\_\_\_\_\_ a year for \_\_\_ years, how do I calculate what the value of that series of investments would be \_\_\_ years from now assuming I earn a compounded interest rate of \_\_\_% on my investments?

This type of problem requires the calculation of the future value of a regular series of payments. Where each payment is made at the beginning of a compounding period (for example, at the beginning of each year), the process is known as an annuity due or an annuity in advance.

B. What if the first payment in my series of investments is not made until one year from now? In this case the process is known as an ordinary annuity or an annuity in arrears.

SOLUTIONS:

A. Go to Appendix E, Future Value of an Annuity Due Table. This table reflects the amount to which $1 deposited at the beginning of each year will accumulate at compound interest for a given number of years at various interest rates. For example, to find out how much $1,000 a year, invested at the beginning of each year, will be worth at the end of 20 years, if the invested annual payments grow at the rate of 10% per year, multiply $1,000 by the factor found in the 20-year row under the 10% column. That factor is 63.0025. Therefore, the investments of $1,000 per year, as of the beginning of each year, would be worth $63,002.50 at the end of 20 years.

This annuity table was compiled using the following formula that can be used in cases not included in the table:

Equation FVAD



In this formula,

*FV* = Future Value Amount

*A* = The Amount of the Annual (Annuity) Investment (Payment)

*n* = The Number of Years (Periods) of Annual Investments (Payments)

*i* = Interest Rate for each Year (Period)

Applying the formula to our example,



In an investment context, a common example of an annuity due is the amount an investor would deposit at the beginning of each year in order to provide funds for retirement at a specified retirement age.

B. Go to Appendix F, Future Value of an Ordinary Annuity Table. This table reflects the amount to which $1 deposited at the end of each year will accumulate at compound interest for a given number of years at various interest rates. For example, to find out how much $1,000 a year, invested at the end of each year, will be worth at the end of 20 years, if the invested annual payments grow at the rate of 10% per year, multiply $1,000 by the factor found in the 20-year row under the 10% column. That factor is 57.2750. Therefore, the investments of $1,000 per year, as of the end of each year, would be worth $57,275 at the end of 20 years.

This annuity table was compiled using the following mathematical formula that can be used in cases not included in the table:

Equation FVOA



In this formula,

*FV* = Future Value Amount

*n* = The Number of Years (Periods) of Annual Investments (Payments)

*A* = The Amount of the Annual (Annuity) Investment (Payment)

*i* = Interest Rate for each Year (Period)

Applying the formula to our example,



In an investment context, an example of an ordinary annuity is the deposits an investor would make at the end of each year in order to provide funds for retirement at a specified retirement age.

It is important to compare the future value of the series of payments made at the beginning of each year ($63,002) with the future value of the series of payments made at the end of each year ($57,275). This $5,727 difference results from the additional compounding on all of the payments.

If interest is equal to 0%, then the future value of an annuity (ordinary or due) equals the sum of all payments, or *A* ∙ *n*.

IV. Computing the Present Value of a Regular Series of Receipts

PROBLEM: If, beginning one year from today, I receive $\_\_\_\_\_\_ a year for \_\_\_ years, how do I calculate the present value of that series of payments, assuming a \_\_\_% discount rate?

SOLUTION: Go to Appendix C, Present Value of an Ordinary Annuity Table. This table reflects the present value of $1 received annually at the end of each year for a given number of years at various discount rates.

For example, to compute the present value of $1,000 received at the end of each year for a 20-year period, discounted at the rate of 10% per year, you would multiply $1,000 by the factor found in the 20-year row under the 10% column. That factor is 8.5136. Therefore, the receipt of $1,000 at the end of each year for the next 20 years has a present value of $8,513.60.

This annuity table was compiled using the following formula that can be used in cases not included in the table:

Equation PVOA

****

In this formula,

*PV* = Present Value Amount

*A* = The Amount of the Annual (Annuity) Receipts

*i* = Interest Rate for each Year (Period)

*n* = The Number of Years (Periods) of Annual Receipts

Applying the formula to our example,

****

The corresponding factor for the present value of an annuity with payments at the beginning of each year can be found in Appendix B, Present Value of an Annuity Due Table. The corresponding formula is:

Equation PVAD



If interest is equal to 0%, then the present value of an annuity (ordinary or due) equals the sum of all payments, or *A* ∙ *n*.

V. Practical Examples

1. PROBLEM: Rich Stevens, age 53, has just inherited $100,000 which he would like to use as part of his retirement nest egg. Rich would like to know just how much the $100,000 will be worth in 12 years, when he will reach age 65, assuming the funds can be invested for the entire period at a 12% annual rate. He would also like to know what the future value of the $100,000 would be in only 7 years, when he reaches age 60, in case he decides to retire early.

SOLUTION: Using the table in Appendix D, the future value of the $100,000 at the end of 12 years and 7 years can be computed as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Present Value** |  | **12% Interest Factor** |  | **Future  Value** |
| 12 Years | $100,000 | x | 3.8960 | = | $389,600 |
| 7 Years | $100,000 | x | 2.2107 | = | $221,070 |

2. PROBLEM: Now that Rich knows how much the $100,000 inheritance will be worth in both cases, he would like to know how much he could withdraw from the fund in equal installments at the end of each year from the year he reaches age 65 until he reaches age 70½, the year he must start withdrawing funds from his individual retirement account (IRA). Rich assumes the funds will continue to earn at a 12% annual rate. In other words, Rich would like to know the annual year-end payment from (1) a 6-year annuity (from age 65 to the year he will be 70½), earning 12% annually on a principal sum of $389,600, and (2) an 11-year annuity (from age 60 to the year he will be 70½), earning 12% annually on a principal sum of $221,070.

SOLUTION: Using the table in Appendix C, the year-end annual annuity payment amounts can be computed as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Principal Sum |  | 12% Annuity Factor |  | Annual Annuity Amount |
| 6-Year Annuity | $389,600 | ÷ | 4.1114 | = | $94,761 |
| 11-Year Annuity | $221,070 | ÷ | 5.9377 | = | $37,232 |

3. PROBLEM: Rich has determined that he will need $60,000 per year from the inheritance fund to handle his living needs until he reaches age 70½. Assuming the fund will continue to earn 12% annually, at what age can Rich afford to retire? (Rich has already decided not to touch his IRA funds until the latest possible date, believing he can cover his living costs with the inheritance until that time. He is even willing to adjust his retirement date by a year or so if need be.)

SOLUTION: This problem is more difficult, but a reasonably accurate answer can be computed using trial and error and the tables in Appendix C and Appendix D.

We have already determined that if Rich waits until age 65 to retire, the inheritance will grow, at a 12% annual interest rate, to $389,600 when he retires. The $389,600 will provide him with a 6-year annual annuity of $94,761, until he reaches age 70½. This annual annuity is $34,761 per year more than the $60,000 Rich believes he needs. Therefore, Rich should be able to retire before reaching age 65.

On the other hand, we have also computed that Rich’s inheritance will grow, at a 12% rate, to only $221,070 by the time he reaches age 60. In this case the resulting annual annuity would be only $37,232 until he reaches age 70½, 11 years after retiring. This annual annuity is $22,768 ($60,000 – $37,232) per year short of Rich’s $60,000 estimated annual need until reaching age 70½. Consequently, it does not appear that Rich can retire as early as age 60.

On a trial and error basis, let’s compute what would happen if Rich retires at age 62. The $100,000 would grow for 9 years at 12% to $277,310, computed as follows, using the table in Appendix D:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Present Value** |  | **12% Interest Factor** |  | **Future  Value** |
| 9 Years | $100,000 | x | 2.7731 | = | $277,310 |

The $277,310 would provide Rich an annuity for 9 years at 12%, until he reaches age 70½, of $52,046, computed as follows, using the table in Appendix C:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Principal Sum** |  | **12% Annuity Factor** |  | **Annual Annuity Amount** |
| 9-Year Annuity | $277,310 | ÷ | 5.3282 | = | $52,046 |

On this basis, Rich falls $7,954 short of reaching his goal of a $60,000 annual annuity if he retires at age 62. Let’s see what happens if he retires at age 63. The $100,000 would grow for 10 years at 12% to $310,580, computed as follows, using the table in Appendix D:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Present Value** |  | **12% Interest Factor** |  | **Future  Value** |
| 10 Years | $100,000 | x | 3.1058 | = | $310,580 |

The $310,580 would provide Rich an annuity for 8 years at 12%, until he reaches age 70½, of $62,521, computed as follows, using the table in Appendix C:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Principal Sum** |  | **12% Annuity Factor** |  | **Annual Annuity Amount** |
| 8-Year Annuity | $310,580 | ÷ | 4.9676 | = | $62,521 |

On this basis, Rich will exceed his goal of a $60,000 annual annuity by $2,521, if he retires at age 63. Therefore, although the tables in Appendix C and Appendix D cannot provide an exact retirement date, they can be used to provide a reasonable approximation. In this case, Rich Stevens can attain his retirement goal of a $60,000 annual annuity, to last until he reaches age 70½, using his $100,000 inheritance, if he retires sometime just before attaining age 63.

4. PROBLEM: Rich has decided that he wants to retire at age 60. He would like to know how much of his other funds need be set aside with his $100,000 inheritance in order to reach his goal of a $60,000 annuity from age 60 until the year he reaches age 70½. Rich assumes the funds can continue to earn at a 12% annual rate.

SOLUTION: The first step in determining the amount that must be added to the $100,000 inheritance is to determine the lump sum needed as of the anticipated retirement date in 7 years (age 60). At age 60, Rich will need $356,262 in order to provide an annual annuity of $60,000 for the 11 years from age 60 until the year he reaches age 70½. This lump sum can be computed as follows, using the table in Appendix C (present value of an annuity):

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Annual Annuity Amount** |  | **12% Annuity Factor** |  | **Principal Sum** |
| 11-Year Annuity | $60,000 | ÷ | 5.9377 | = | $356,262 |

In order for Rich to accumulate $356,262 by the time he retires at age 60, he must currently invest, at a 12% annual rate, a lump sum amount of $161,137, which can be computed in the following manner, using the table in Appendix A:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Future Value** |  | **12% Annuity Factor** |  | **Present Value** |
| 7 Years | $356,262 | ÷ | 0.4523 | = | $161,137 |

Because Rich has $100,000 from his inheritance, he must add $61,137 in order to accumulate the $356,262 needed to fund an 11-year, $60,000 annuity, to begin in 7 years, when Rich reaches age 60.

5. PROBLEM: Suppose Rich Stevens does not have $61,137 of other funds? How much must he set aside at the beginning of each year over the next seven years, together with the $100,000 lump sum from his inheritance, to reach his $356,262 objective at age 60?

SOLUTION: The $100,000 will grow to $221,070 in the 7 years until Rich reaches age 60, assuming a 12% annual interest rate (see the solution to Problem 1 for this computation). The shortfall at age 60 would be $135,192, the difference between the $356,262 needed and the $221,070 compounded from the original $100,000 inheritance. The $135,192 shortfall can be funded with annual investments of $11,964 computed as follows, using the table in Appendix E:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Future Value** |  | **12% Interest Factor** |  | **Annual Investment** |
| 7 Years | $135,192 | ÷ | 11.2997 | = | $11,964 |

TAXES, INFLATION, AND GROWTH

At times it will be necessary to factor taxes, inflation, and growth into the time value of money concepts of present and future values. Taxes and inflation may reduce the time value of an investment. Sometimes, annuity payments may be determined in a way that takes into account growth in payments, perhaps to reflect inflation.

I. Taxes

Taxes reduce the effective rate of return on investments. However, different investments may be subject to tax at different times. Also, different taxpayers and different investments may be subject to different rates of tax. Furthermore, tax rates may be different in different years. All other things being equal, a higher tax rate reduces the rate of return more than a lower tax rate.

If an investor is taxed annually on investment income, the rate of return can be adjusted to an after-tax rate of return by multiplying the rate of return by one minus the investor’s tax rate. For example, an investor earns 8% before tax and is subject to a 25% tax on such earnings. The investor has an after-tax rate of return of 6% [8% ∙ (1 – 25%)]. This after-tax rate of return could be used as the rate of return in any of the present and future value formulas above.

If the investor is taxed only at some later time (i.e., tax is deferred), perhaps on distribution or disposition, the investment could be valued using a before-tax rate of return with tax subtracted at the later time. For example, an investor makes a deductible (before-tax) contribution of $2,000 to a traditional IRA. The IRA contribution grows at an 8% before-tax rate of return to $10,000, when it is distributed. If the distribution is subject to a 25% tax, the investor is left with $7,500 [$10,000 ∙ (1 – 25%)]. [The investor would also have received a tax deduction at the time of contribution worth $400 ($2,000 ∙ 25%) which could also be invested.]

Assume instead that the investor makes a nondeductible (after-tax) contribution of $2,000 to a traditional IRA. The IRA contribution grows at an 8% after-tax rate of return to $10,000, when it is distributed. If the distribution is subject to a 25% tax, the investor is left with $8,000 [$2,000 + ($10,000 - $2,000) ∙ (1 – 25%)].

When capital gains are factored in and taxed periodically, while dividends are received and taxed annually, the calculation of present and future values can become quite complex. Generally, the way to make such complex calculations is to break the valuation down into its separate parts. A spreadsheet is often useful.

II. Inflation/Growth

Often investment analyses and other financial planning problems involve adjustments for inflation or for expected systematic increments or decrements of payments or cash flows over time. For example, when planning for how much one must accumulate for retirement, it is common to assume that the amount needed each year in retirement will increase as a result of inflation. The annuity formulas presented earlier will compute the present value of a series of level payments for a specified number of years, but how does one compute the present value if the payments are assumed to be increasing at some constant rate rather than remaining level?

Actually, the formulas given above are generally still perfectly applicable, with some slight modification, if one substitutes inflation-adjusted or growth-adjusted rates for nominal rates.

The inflation- or growth-adjusted rate of return, *ρ*, is defined as follows, where *r* is the nominal rate of return and *g* is the inflation or *g*rowth factor:

Equation RIA



For example, an investor earns 12% on her investment for the year. However, inflation for the year is 4%. Her real inflation-adjusted rate of return is calculated as follows.



*Example.* A client’s child will be attending college in 5 years. The client wants to know how much she will need to set aside today to pay the first year’s tuition and fees. Assume current tuition and fees are $36,000, and inflation for college costs averages 6%, and she can earn 5% on the money she invests for this purpose. One can compute the amount needed directly using just the present value equation PVLS above with the inflation-adjusted rate of return of –0.9434% [(5% - 6%) ÷ (1 + 6%)] (*ρ*), 5 years (*n*), and the $36,000 goal (*FV*). She will need to invest $37,747 today.





III. Inflation/Growth Adjusted ROR and Annuities

The inflation/growth-adjusted rate becomes much more useful when dealing with calculations involving annuities. For inflation-adjusted ordinary annuities where one assumes the payments grow at *g*% per period, the present and future values can be computed by adjusting the annuity formulas for the growth of payments. Annuities with such fixed adjustments for inflation are sometimes referred to as serial payments. The inflation/growth adjusted formulas for such annuity payments follow.

Equation PVOAg

****

****

Equation PVADg

****

****

Equation FVOAg

****

****

Equation FVADg

****

****

*Example.* A client’s child will be attending college in 5 years. The client wants to know how much she will need to set aside today to pay four years of tuition and fees. Assume current tuition and fees are $36,000, and inflation for college costs averages 6%, and she can earn 5% on the money she invests for this purpose. She will need to invest $153,160 today.

One way to calculate the amount required derives from realizing that the present value of a 4-year annuity due commencing at the end of 5 years is equal to the value of a 9-year annuity due commencing today less the value of a 5-year annuity due commencing today.

Using equation PVADg with *n* = 9, *ρ* = –0.9434%, and *Pmt* = $36,000 (use the current college cost value because the analysis starts from today), the present value of a 9-year inflation-adjusted annuity is $336,621. Similarly, using equation PVADg with *n* = 5, *ρ* = –0.9434%, and *Pmt* = $36,000, the present value of a 5-year inflation-adjusted annuity is $183,461. The difference is $153,160 ($336,621 - $183,461).

NET PRESENT VALUE AND INTERNAL RATE OF RETURN

Often, an investment will consist of one of more amounts paid by the investor (negative cash flow) and one or more amounts received by the investor (positive cash flow). Furthermore, the cash flows may be irregular, with different amounts paid or received in different years. Net present value calculations apply the present value building blocks above to calculate a net present value for such negative and positive cash flows. Alternatively, one can use internal rate of return calculations to estimate what an investment with negative and positive cash flows would earn, building on the same present and future value building blocks. The following discussion also addresses a couple of other simple methods for evaluating investments.

Investors can compare alternative investments using the following methods.

1. *Net Present Value* (*NPV*) is the difference between the present value of all future benefits of an investment and the present value of all capital contributions. This method measures the tradeoff between the cash invested and the benefits projected.

2. *Internal Rate of Return* (*IRR*) is that rate at which the present value of all the future benefits an investor will receive from an investment exactly equals the present value of all the capital contributions the investor will be required to make. IRR generally is used to compare the *effective* interest rates of two or more investments.

3. *Adjusted Rate of Return (ARR)* is often called effective rate of return. Investors calculate the *ARR* by assuming that they will invest all of the investment’s benefits (not only cash inflows but also tax savings) at the alternative reinvestment rate. The alternative reinvestment rate is the after-tax rate at which they can safely invest the money.

4. *Payback Period* (*PP*) measures the relative periods of time needed to recover the investor’s capital (income received after the payback period will be considered gain).

5. *Cash On Cash (COC)*, as its name implies, analyzes an investment by dividing the annual cash flow by the amount of the cash investment in order to determine the cash return on the cash invested.

I. Net Present Value

*Net Present Value* (*NPV*)is an extension of the present value concepts discussed above. Present value is the amount that one must invest now to produce a given future value. For instance, if I assume I can invest money at 10%, I must have $1,000 now in order to have it grow to $1,100 one year from now. $1,000 is the present value of $1,100 to be received in one year. Obviously, the present value is affected by: (1) the interest (investment analysts call this the discount) rate; as well as (2) the length of the investment period.

Present value is a simple means of comparing two investments. For example, I am considering an investment of $1,000 that will pay me $1,200 three years from now. I can also invest the $1,000 in an alternative investment of equal risk and earn 10% on my money. Which investment should I make?

An easy way to compare the investments is to compute the present value of the $1,200 payable three years from now at a 10% discount rate. The present value of the first investment is only $902, while obviously the present value of $1,000 in hand today is $1,000. Therefore, from a pure present value standpoint, the proposed investment is inferior to the alternative of simply investing the $1,000 at 10%.

*NPV* is the difference between: (1) the present value of all future benefits to be realized from an investment; and (2) the present value of all capital contributions into the investment. A negative *NPV* should result in an almost automatic rejection of the investment. A positive *NPV* indicates that the investment is worth further consideration because the present value of the stream of dollars the investor will recover exceeds the present value of the stream of dollars the investor will have to pay out.

The difficulty with this method arises with respect to what discount rate one should use in computing the present values of the cash inflows and cash outflows? Usually this discount rate will be the minimal acceptable rate of return. One usually finds this is rate by determining the cost of capital or, as in the example above, determining the rate an alternative investment of similar quality can earn. In the example above, the rate was 10%. Once one determines this so-called reinvestment rate, one can use it as the discount factor to compute the present value of the money invested and the present value of the expected return.

Once one computes these present value amounts, one then nets them against each other. If the result is positive, the investment will exceed the reinvestment rate and one should seriously consider making the investment. If the net present value is a negative number and falls short of the reinvestment rate, the one generally reject the investment under consideration.

To the extent that a proposed investment yields a positive net present value, the investment provides a potential cushion for safety. It also may allow the investor to incur certain additional costs, such as attorney’s, accountant’s, or financial planner’s fees in connection with the analysis of the investment, and still achieve the desired reinvestment rate.

An example of the use of net present value analysis may be helpful.

Assume that at the beginning of the year an individual has been shown an investment opportunity requiring a lump sum outlay of $10,000. Currently the funds he would use for this investment are in a bond fund earning 6% annually, net after taxes. The investment proposal projects the following after-tax cash flows at the end of each year.

|  |  |
| --- | --- |
|  | **Amount** |
| Year | **Received** |
| 1 | $ 2,000 |
| 2 | 1,500 |
| 3 | 750 |
| 4 | 500 |
| 5 | 10,000 |
| Total Receipts | $14,750 |

Based solely on net present value analysis, should the investor make this investment?

The first step in the decision analysis is to determine the appropriate reinvestment rate. In this example, the investor currently is earning a net after tax return of 6% in what he believes is a safe investment. To warrant any further consideration, this proposed investment must have a positive net present value when using the benchmark reinvestment rate of 6% as the discount rate.

Does the proposed investment meet that benchmark? The stream of dollars projected to be received from the proposed investment has a present value of $11,720 assuming a 6% discount rate. The net present value is a positive $1,720 (the difference between the present value of the future stream of cash inflows, $11,720, and the $10,000 present value of the lump sum outflow of $10,000). Therefore, the proposal does deserve further consideration.

But what if the investor demands a rate of return from the proposed investment higher than his benchmark rate of 6% in order to compensate for the additional risk? If he sets a 15% rate as his minimum, the present value of the stream of dollars from the proposed investment is only $8,624. The net present value is a negative $1,376, the difference between the present value of the future stream of cash inflows, $8,624, and the $10,000 present value of the lump sum outflow of $10,000. Therefore, the proposal does not deserve further consideration.

What discount rate when applied to the expected stream of cash inflows from the proposed investment has a present value exactly equal to the $10,000 lump sum investment? That discount rate is 10.5%. This computation illustrates the concept of Internal Rate of Return, discussed more fully below.

How to Compute Net Present Value

**A. Lump Sum Investment, Single Future Receipt**

Assume an individual makes a lump sum investment at the beginning of year one of $10,000. The expected return on this investment is $15,000 (after tax) to be received as a single amount at the end of year five. The investor’s discount rate, for an alternative safe investment, is 6% after tax. What is the net present value of the investment under consideration?

To compute the net present value of the investment, the following basic steps are necessary.

1. Compute the present value of the $10,000 investment. Because only one payment is required, immediately at the beginning of the cash flow period, the present value of that payment would be $10,000.

2. Compute the present value of the $15,000 future amount to be received, at the end of year five, using the 6% discount rate. Refer to the Present Value Table in Appendix A. The applicable factor for the present value of $1 at the end of 5 years, using a 6% discount rate, is 0.7473. One multiplies this factor by the $15,000 amount to be received in the future. The present value therefore is $11,210 ($15,000 ∙ 0.7473).

3. Subtract the present value of the $10,000 lump sum investment ($10,000) from the present value of the $15,000 single payment to be received ($11,210). The net amount is +$1,210; a positive net present value. Note that if the $15,000 were not received until the end of seven years, the present value of the receipt would be only $9,977 ($15,000 ∙ 0.6651 discount factor), resulting in a “negative” net present value of $23.

**B. Lump Sum Investment, Multiple Future Receipts**

Assume an individual makes a lump sum investment at the beginning of year one of $10,000, the present value of which is $10,000 [Step (1)].

The expected return on this investment (received at each year-end) and the present value of each receipt, discounted at 6% are as follows [Step 2].

|  |  |  |  |
| --- | --- | --- | --- |
| **Year** | **Amount Received** |  | **PV  @ 6%** |
| 1 | $ 2,000 |  | $ 1,887 |
| 2 | 1,500 |  | 1,335 |
| 3 | 750 |  | 630 |
| 4 | 500 |  | 396 |
| 5 | $10,000 |  | $7,473 |
| Total Receipts | $14,750 |  | $11,721 |

The present value amounts in this table were computed using the Present Value Table in Appendix A. Computations made using different tables or software may vary slightly due to rounding differences, including the number of decimal places to which factors are rounded.

The net present value therefore is $1,721 ($11,721 present value of the future flow of receipts, less $10,000 present value of the lump sum investment) [Step 3].

**C. Multiple Investments, Multiple Future Receipts**

Continuing the above example, assume that instead of one $10,000 investment at the beginning of year one, there will be two $5,000 payments, one at the beginning of year one, and the other at the beginning of year two. The present value of the investment, using the same 6% discount rate would be computed as follows [Step 1].

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Year** | **Payment** |  | **@ 6%** |
| [Beginning] | 1 | $ 5,000 |  | $ 5,000 |
| [Beginning] | 2 | 5,000 |  | 4,717 |
| Total Payments |  | $10,000 |  | $ 9,717 |

When an investment requires multiple pay-ins over a period of time, one must determine the present value of these payments in determining the advisability of the investment. In this example, the present value of the first $5,000 pay-in is $5,000. The present value of the second pay-in is of $5,000 is $4,717, $5,000 multiplied by the present value factor of 0.9434 (because the payment is made at the beginning of the second year, the present value factor for the end of the first year is appropriate). If one assumes that the present value of the receipts are the same as above, $11,721 [Step 2], then the net present value of the investment with this revised pay-in stream is $2,004 ($11,721 - $9,717) [Step 3].

II. Internal Rate of Return (*IRR*)

When the concept of Net Present Value (*NPV*) was introduced above, it was defined as the difference between the present value of all future benefits to be realized from an investment and the present value of all capital contributions into the investment. The discount (interest) rate at which these two present values will be equal is the Internal Rate of Return (*IRR*) of the investment.

Stated in other terms, in computing *IRR* the interest rate sought is that rate at which inflows and outflows of cash, discounted to present value, will equal the original principal. It is a method of determining what percentage rate of return cash inflows will provide based on a known investment (cash outflow) and estimated cash inflows.

Internal rate of return really is the same as a present value computation except that the discount rate is either not known or not given. The financial adviser therefore is attempting to find that rate which will discount the future cash inflows so that they will precisely equal the investor’s initial investment.

Confused? Let’s try an example.

Assume your client is considering the purchase of a $100,000 unit of a limited partnership. The full $100,000 is due at the beginning of the year. You have estimated that, after taxes, she should be receiving the following cash inflows at the end of each year.

|  |  |
| --- | --- |
| **End of  Year** | **In-Flow** |
| 1 | $ 10,000 |
| 2 | 10,000 |
| 3 | 120,000 |

If we were to do a *NPV* analysis of this investment, looking at several alternative rates of return, it would look as presented in Figure 20.1.

**Figure 20.1**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Outflows**  **Begin of Yr** | **Amount  Paid** | **Present Value of Amount Paid @** | | | |
| **8%** | **10%** | **12.9%** | **20%** |
| Year 1 | -$100,000 | -$100,000 | -$100,000 | -$100,000 | -$100,000 |
|  |  |  |  |  |  |
| **Inflows**  **End of Yr** | **Amount Received** | **Present Value of Amount Received @** | | | |
| 8% | 10% | 12.9% | 20% |
| Year 1 | $ 10,000 | $ 9,259 | $ 9,091 | $ 8,857 | $ 8,333 |
| 2 | 10,000 | 8,573 | 8,264 | 7,845 | 6,944 |
| 3 | 120,000 | 95,260 | 90,158 | 83,387 | 69,444 |
|  |  |  |  |  |  |
| Total of *PV* of Inflows | | $113,092 | $107,513 | $100,089 | $84,721 |
|  |  |  |  |  |  |
| Net Present Value (*NPV*) | | $ 13,092 | $ 7,513 | $ 89 | -$15,279 |

At an 8% discount rate, this investment has a positive net present value of $13,092. That is, if you had invested $100,000 at 8% (for example, in a certificate of deposit) the present value of the future cash inflow should be $100,000. But, in the investment above, the present value of the expected cash inflows is actually $113,092, $13,092 higher than it should be at 8%. Therefore, it is obvious that the investment is generating a significantly higher rate of return than 8%.

At a 10% discount rate, this investment has a positive net present value of $7,513. That is, if you had invested $100,000 at 10% the present value of the future cash inflow should be $100,000. But, in the investment above, the present value of the expected cash inflows is actually $107,513, $7,513 higher than it should be at 10%. Therefore, it appears that the investment is generating a higher rate of return than 10%.

At a 20% discount rate, this investment has a negative net present value of $15,279. That is, if you had invested $100,000 at 20% the present value of the future cash inflow should be $100,000. But, in the investment above, the present value of the expected cash inflows is actually $84,721, $15,279 lower than it should be at 20%. Therefore, it is obvious that the investment is generating a lower rate of return than 20%.

What is the actual rate of return on this investment? As you can likely figure out, it is somewhere between 10% and 20%. Internal rate of return (*IRR*) is a method of computing that break-even rate. The chart in Figure 20.1 illustrates that the internal rate of return of the investment is approximately 12.9%. That is, the present value of the cash outflows ($100,000) is roughly equal to the present value of the cash inflows ($100,089) when discounted at a 12.9% rate.

How to Compute Internal Rate of Return

In manually computing the *IRR* of this investment the first step is to compute the *NPV* using a preliminary estimate of the *IRR*. If the first computation results in a positive *NPV*, a second calculation, using a higher discount rate, will be necessary. If a negative *NPV* is computed, the recalculation will require a lower discount rate. The process would have to continue until you arrive at a *NPV* of $0 (i.e., the rate at which the present value of the cash outflows equals the present value of the cash inflows).

If we use 8% as our initial (test) discount rate, we find a positive net present value, and therefore must try a higher discount rate. On our second attempt, using 10%, we still have a positive *NPV*. Our third computation, using 20%, yields a negative *NPV*. Therefore, the *IRR* must be between 10% and 20%.

After several attempts, we finally try a rate of 12.9% that results in a positive *NPV* of only $89. We’re getting close. For most planning purposes, this would be close enough. To do the job more accurately and efficiently, we could use a business calculator or a computer.

Shortcomings of the *IRR* Method

What are the shortcomings of the *IRR*? There are several, but to some extent the shortcomings arise from widespread misconceptions and misunderstandings about the theoretical underpinnings as well as the proper application of the method, even among the most elite and sophisticated investors and investment advisers. Consequently, although *IRR* is one of the most commonly used tools, it is also the least understood and most misapplied method of evaluating alternative investments.

One of the most common misconceptions, even appearing in many, if not most, of the finance and investment textbooks used by our country’s leading business schools, is that the *IRR* method inherently assumes that the cash flows from an investment being evaluated are implicitly reinvested at the computed internal rate of return of the investment itself. This is decidedly *not* the case, but this generally held misconception is one of the principal reasons the *IRR* is often misapplied.

The ***fact*** is that the *IRR* assumes that the cash flows are not reinvested … at any rate. Rather, it assumes cash flows from an investment are consumed when paid and never enter the analysis again. If it were assumed they are reinvested at the IRR, they would not be, or should not be, shown as cash outflows but rather as *reinvested cash inflows*!

The reality is that the *IRR* measures the rate of return on the *unrecovered investment* over time. Cash flows from an investment represent money taken out of the investment. Over time, only the investor’s, as yet, unrecovered investment implicitly continues to earn the *IRR*. The following example shows five alternative investments and demonstrates the point. In each case: only the unrecovered investment earns the *IRR*; cash flows from the investments do not earn anything.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Project Cash Flows** | | | | | |
| **Year** | **A** | **B** | **C** | **D** | **E** |
| 0 | ($1,000) | ($1,000) | ($1,000) | ($1,000) | ($1,000) |
| 1 | 100 | 50 | (200) | 200 | 600 |
| 2 | 100 | 50 | (200) | 200 | 600 |
| 3 | 1,100 | 1,215 | 1,793 | 869 | (55) |
| **IRR** | 10% | 10% | 10% | 10% | 10% |

Each project has a 10% *IRR*.

Project A earns 10%, but also pays out $100, or 10% of the initial $1,000 investment, each year. Therefore, the unrecovered investment earning 10% remains at $1,000 each year, which is similar to a 10% coupon bond purchased at par value.

Project B, in contrast, is similar to a bond that the investor has purchased at a discount. It earns 10% on the initial $1,000 investment, but it only pays out $50 each year. Therefore, Project B’s unrecovered investment increases each year.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Unrecovered Investment** | **Earnings @ 10% IRR** | **(Payout) Pay in** | **Increase (Decrease) In Unrecovered Investment** |
| 0-1 | $1,000 | $100.00 | $50.00 | $50.00 |
| 1-2 | 1,050 | 105.00 | 50.00 | 55.00 |
| 2-3 | 1,105 | 110.50 | 1,215.50 | (1,105.00) |
| 3 | 0 |  |  |  |

Project B is the same as Project A with all but $50 of the yearly cash flows essentially reinvested in the project at 10%. These implicitly reinvested earnings increase the unrecovered investment and earn the 10% *IRR* rate.

Project C has an even greater increasing unrecovered investment over time.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Unrecovered Investment** | **Earnings @ 10% IRR** | **(Payout) Pay in** | **Increase (Decrease) In Unrecovered Investment** |
| 0-1 | $1,000 | $100.00 | $200.00 | $300.00 |
| 1-2 | 1,300 | 130.00 | 200.00 | 330.00 |
| 2-3 | 1,630 | 163.00 | (1,793.00) | (1,630.00) |
| 3 | 0 |  |  |  |

Because none of the earnings are taken out of Project C until the end, all of the implicit earnings are essentially reinvested in the project at 10%.

Project D has a decreasing unrecovered investment over time.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Unrecovered Investment** | **Earnings @ 10% IRR** | **(Payout) Pay in** | **Increase (Decrease) In Unrecovered Investment** |
| 0-1 | $1,000 | $100.00 | ($200.00) | ($100.00) |
| 1-2 | 900 | 90.00 | (200.00) | (110.00) |
| 2-3 | 790 | 79.00 | (869.00) | (790.00) |
| 3 | 0 |  |  |  |

In this case, the project throws off cash flows that are more than the implicit earnings each year, thereby reducing the unrecovered investment over time and, as a result, reducing the implicit earnings.

Project E has decreasing and, ultimately, negative unrecovered investment over time.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Unrecovered Investment** | **Earnings @ 10% IRR** | **(Payout) Pay in** | **Increase (Decrease) In Unrecovered Investment** |
| 0-1 | $1,000 | $100.00 | ($600.00) | ($500.00) |
| 1-2 | 500 | 50.00 | (600.00) | (550.00) |
| 2-3 | (50) | (5.00) | 55.00 | (50.00) |
| 3 | 0 |  |  |  |

One can actually view Project E as a combination of investment and loan because it requires the investor to contribute additional dollars at the end to pay back early withdrawals in excess of his remaining unrecovered investment. Because there are no explicit payments on the loan, except to pay off the entire amount at the end, the implicit loan does, in fact, charge interest at a rate equal to the 10% *IRR*.

Although it is true that *if* the cash flows were reinvested at the *IRR*, the computed *IRR* would not change, this misconception about the *IRR* and reinvestment of cash inflows leads to errors in the application of the method. In some cases, it does not matter at what rate an investor can reinvest cash flows; in other cases, it makes all the difference in the world.

Specifically, when investors want to know what rate of return they will earn on an investment, their potential reinvestment rate for the cash flows is immaterial, generally. The *IRR* is the rate of return they will earn on the investment itself.

However, when investors want to use the *IRR* to compare investments that involve different initial outlays, cash-flow patterns, and/or investment terms, they must explicitly account for the differences in cash flows, otherwise they, not the *IRR* method, are implicitly assuming cash flows are reinvested at the *IRR*. When reinvestment at the *IRR* is not a realistic possibility, the *IRR* method, improperly used, leads to poor choices among investments.

For instance, an investor is offered an immediate annuity paying $5,092.61 per year for 20 years for an initial investment of $50,000. He computes the *IRR* and finds that it is 8%. Whether he spends the cash flows or reinvests them is immaterial; the *IRR* method does not care. Whatever rate he might earn by reinvesting the cash flows paid to him does not change the return on the investment in the annuity. The annuity is paying 8%, and that is a fact regardless of whether or not and at what rate he reinvests those cash flows.

Now, however, assume the investor also is offered another annuity that will pay him $6,021.02 per year for 15 years for the same $50,000 initial investment. He computes the *IRR* for this annuity and finds that it is 8.5%. Is this the better deal? Not necessarily, even though it is, in fact, paying a half a percent more on his investment.

The 8.5%, 15-year annuity might be the better deal if he could reinvest part or all of the $928.41 annual difference in cash flows for the first 15 years so that he could at least match the cash flows he would otherwise receive for the last five years of the 8%, 20-year annuity. But if he cannot, the 8% annuity would be the better deal, even though it really is paying one-half percent less on his money.

Assume the investor feels he could earn 5% by reinvesting the difference in cash flows in an investment with the same level of safety and security as the annuity. The $928.41 annual difference invested each year at 5% would grow to $20,033.81 in 15 years. If this amount continues to earn 5%, he could withdraw $4,627.31 each year for the next 5 years before exhausting the fund. But that is $465.30 per year less than he would receive from the 8%, 20-year annuity. So he would be worse off investing in the 8.5%, 15-year annuity than if he invested in the 8%, 20-year annuity.

Obviously, if his best alternative investment with the same level of safety and security earns only 5%, then both of these annuities are good investments for him. The *IRR* of either one is greater than his opportunity cost rate or hurdle rate of 5%. If he had $100,000 to invest and was limited to a $50,000 investment in each annuity, he should invest in both of them. However, if he ranked the annuities based on their *IRRs* in order to choose between them, he would have made the wrong choice.

In addition to the *IRR* method often being misunderstood and misapplied, the *IRR* method has other real weaknesses, which one can overcome with proper use of the method. The weaknesses include

* As described above, investors cannot use the unmodified *IRR* method to compare (directly) mutually exclusive investments, the investment with the highest *IRR* is not necessarily the best investment among a mutually exclusive set of investment opportunities.
* The unmodified *IRR* method does not consider realistic reinvestment rates for positive cash flows or realistic borrowing rates for negative cash flows over the holding period.
* An investment project may have multiple *IRR*s.
* Solving for the *IRR* often requires a series of iterative calculations to successively home in on the *IRR* because, for many types of *IRR* calculations, there is no single, closed-end formula to compute the *IRR*. However, financial calculators and computer software programs, such as the ubiquitous spreadsheet programs, have built-in functions that are adequate to solve for the *IRR* in most cases.

Let us start with the problem of multiple *IRR*s. The other issues will be discussed in later sections of this chapter.

How can an investment have more than one *IRR*?

Let us prove it first and then explain why. Figure 20.2 shows an admittedly odd hypothetical investment that has four *IRR*s. The investment involves an initial outlay of $1,000, a pay-in of $4,700, another outlay of $8,277.50, another pay-in of $6,356.75, and then one last payout of $1,828.78. As the table shows, if one discounts this cash flow stream at -5%, 10%, 25%, and 40%, in each case the *NPV* is zero. By definition, an *IRR* is a discount rate that equates the net present value of the cash flows to zero, so these rates are all *IRR*s.

Potentially, an investment has as many different real *IRR*s as there are changes in the sign, or direction, of the cash flows over time. In most cases when there is a single initial investment (a payout), or a series of investment payouts, and, subsequently, a series of cash pay-ins, there is only one distinct *IRR*, because there is only one change in the direction or sign of cash flows. However, if the investment involves a series of payouts mixed over time with a series of pay-ins, then each time the cash flow stream changes from payouts to pay-ins, or vice versa, there could potentially be more *IRR*s.

One problem with multiple *IRR*s is that it is not always easy to find them all and most software solutions will find just one, or refuse to solve for any at all if there could be more than one *IRR*. Regardless of the initial guess or seed one picks to start the search for multiple *IRR*s, the calculator or software program may home in on a particular *IRR*, even if it happens to be the one furthest away from the initial guess. Generally there is no assurance that picking a seed closer to one of the *IRR*s than the others will cause the solution algorithm to find the nearest *IRR*. Consequently, even if you know that there is more than one *IRR*, after you find one, changing your guess or seed to find the next *IRR* will not always be successful. Finding each successive *IRR* tends to become increasingly difficult.

In addition, just because there could be several distinct real *IRR*s does not mean that there actually are several distinct real *IRR*s. In some cases, some of the *IRR*s are in the realm of imaginary numbers: those odd numbers that involve the square root of -1. Although such numbers have real-world significance in the arena of physics and electronics, for example, they have no practical significance in the realm of finance and investments.

**Figure 20.2**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **MULTIPLE INTERNAL RATES OF RETURN** | | | | | |
|  | **Period 0** | **Period 1** | **Period 2** | **Period 3** | **Period 4** |
|  |  |  |  |  |  |
| Cash Flow | (1,000.00) | 4,700.00 | (8,227.50) | 6,356.75 | (1,828.78) |
|  |  |  |  |  |  |
| Interest -5% | 1.00000 | 1.05263 | 1.10803 | 1.16635 | 1.22774 |
| PV CF | (1,000.00) | 4,947.37 | (9,116.34) | 7,414.22 | (2,245.23) |
| Cum. PV CF | (1,000.00) | 3,947.37 | (5,168.98) | 2,245.23 | 0.00 |
|  |  |  |  |  |  |
| Interest 10% | 1.00000 | 0.90909 | 0.82645 | 0.75131 | 0.68301 |
| PV CF | (1,000.00) | 4,272.73 | (6,799.59) | 4,775.92 | (1,249.06) |
| Cum. PV CF | (1,000.00) | 3,272.73 | (3,526.86) | 1,249.06 | 0.00 |
|  |  |  |  |  |  |
| Interest 25% | 1.00000 | 0.80000 | 0.64000 | 0.51200 | 0.40960 |
| PV CF | (1,000.00) | 3,760.00 | (5,265.60) | 3,254.66 | (749.06) |
| Cum. PV CF | (1,000.00) | 2,760.00 | (2,505.60) | 749.06 | 0.00 |
|  |  |  |  |  |  |
| Interest 40% | 1.00000 | 0.71429 | 0.51020 | 0.36443 | 0.26031 |
| PV CF | (1,000.00) | 3,357.14 | (4,197.70) | 2,316.60 | (476.04) |
| Cum. PV CF | (1,000.00) | 2,357.14 | (1,840.56) | 476.04 | 0.00 |

Even if one finds all the real *IRR*s, the question remains: which is the right one? Mathematically, they are all correct, but which is the correct one financially? Suppose you find that an investment has two *IRR*s, -10% and +25%. If you invest in the project are you losing 10% or making 25%? Perhaps both? Could you be both making 25% and losing 10%, thereby averaging 7.5%? Hardly!

The only way to solve the dilemma is to use one of the modified *IRR* methods discussed in the next section. Depending on how you actually will handle the cash flows from the project, you can determine which *IRR* is in fact closer to the rate you will actually earn on the investment. The added bonus of learning about modified *IRR* methods is that they are also necessary whenever you want to compare or rank investment alternatives, one against another.

III. Modified Internal Rate of Return Methods

As noted above, at times it may be useful to apply a different rate of return to amounts distributed from an investment. Or a different rate of return might be used where some additional investment is required in later years after the initial investment. The question arises: what interest rate should be used for the amounts while they are held outside the investment being analyzed?

One rate of return that is useful is the safe rate of return. This is the rate of return that could be earned on a risk-free investment such as treasury bills. Another rate of return that could be used is the reinvestment rate that the investor could obtain in other investments. Still another rate of return is the rate at which the investor could borrow the money. Depending on how the cash flows from the project will actually be handled, one can determine which *IRR* is in fact closer to the rate that will actually be earned on the investment.

IV. Pay Back Period

Pay-back period analysis is a time value of money concept. This method compares alternative investments by measuring the length of time required to recover the original investment. From this perspective, the investment that returns the original capital in the shortest period of time is the best investment.

The major flaw in this analytical technique is obvious: taken to its extreme, an indiscriminate investor would choose a deal requiring a $10,000 investment which paid back $15,000 in one year rather than an alternative which required the same capital outlay but returned $25,000 in two years.

V. Cash on Cash

Cash-on-cash analysis, as its name implies, focuses on the amount of cash generated by the investment. It ignores both taxes and the potential gain from any sale.

To compute cash on cash return, divide the annual cash flow by the cash investment. For example, a woman plans to invest $10,000 in the stock of Z-Rocks Corp. Each year she would receive a dividend of $1,000, and expects to receive $15,000 upon the sale of the stock in three years.

She is considering an alternative investment for the $10,000, Eye-B-Em Stock, which yields a cash distribution of only $600 per year, but which she believes will be worth $25,000 at the end of the three-year investment period. Under the cash-on-cash method, her return on the first investment is 10% per year ($1,000 dividend divided by the $10,000 investment), while the return on the alternative is only 6% per year ($600 divided by the $10,000 investment).

Using only the cash-on-cash method to evaluate the investments, she would choose the Z-Rocks stock, because of the higher annual dividend. However, if she had compared the investments by looking at their relative internal rates of return or adjusted rates of return, she would probably have chosen the alternative.

RISK, PROBABILITIES, AND MODELING

Investments are subject to numerous risks. In addition, financial planning involves other risks, such as that a client may die or become disabled. We will look briefly at risk and modeling for the probability of risk here in the context of time value of money. However, an extensive examination is beyond the scope of this book. For an extensive look at risk and the measurement and handling of risk, see the *Tools & Techniques of Investment Planning*.

Without careful planning, the death of a client may leave the client’s financial plan incomplete. The client’s earnings, or perhaps other sources of income, may stop at death. However, the client may wish to provide for others after his death, including general support for living, or more specialized needs such as for education funding. Life expectancy often is used as a substitute for the probability of death, or how long a person may continue to live, in financial planning. Time value of money calculations often use life expectancy as a period of years. And, of course, life insurance specifically is targeted at such risk.

The disability of a client may also leave the client’s financial plan incomplete. The client’s earnings, or perhaps other sources of income, may also stop at disability. And, once again, the client may still wish to provide for others despite his disability. However, with a disability, the client continues to have expenses, perhaps even increased, after the disability. Disability often is overlooked in financial planning. However, the probability of disability at a particular age often is significantly greater than the probability of death. Time value of money calculations probably also should take into account the probability of disability. And, of course, disability insurance is specifically targeted at such risk.

Investments are subject to an endless list of risks: market, financial, interest rate, business, industry, country, default, inflation, and so on. The rate of return or the range of rates of returns, used in time value of money calculations should reflect such risks. The management of investment risks often involves diversification, asset allocation, and professional management.

In making time value of money calculations, one might use a number of rates of returns to reflect the range of returns that one might expect given all the probabilities that any particular rate of return might be achieved. One might assign probabilities based on standard deviations, a measure of how much actual returns deviate from the average return. A Monte-Carlo simulation on a computer might attempt to take all these probabilities into effect.

Monte-Carlo Simulation

Expected returns and standard deviation are not necessarily constant over time. Nor is the return in an individual period predictable. Returns in each asset class are probabilistic. Investors may also make periodic deposits and withdrawals, further increasing the difficulty in predicting long-term returns and risk. If investors desire certain levels of retirement income, what is the assurance that they are likely to obtain this level of income?

Monte-Carlo simulation is the process of assessing the likelihood of an expected outcome. In a Monte-Carlo simulation, the analyst uses a computer program to randomly choose returns from an expected distribution of returns for each period; perhaps, rebalancing the asset allocation as required under the strategy. Each run results in an ending expected portfolio value. The process is run many times (perhaps 1,000 to 10,000 times), to achieve a distribution of ending values. This process can help the analyst assess the probability of achieving a certain value or income in the future, including the possibility that an individual will outlive his retirement assets.

WHERE CAN I FIND OUT MORE?

1. Leimberg et al, *Tools & Techniques of Investment Planning*, 2nd Edition (Cincinnati, OH: National Underwriter Company, 2006).

2. Fabozzi, *Fixed Income Mathematics: Analytical and Statistical Techniques* (Chicago, IL: Irwin Professional Publishers, 1996).

3. Spaulding, *Measuring Investment Performance: Calculating and Evaluating Investment Risk and Return* (New York, NY: McGraw-Hill, 1997).

4. Rachlin, *Return on Investment Manual: Tools and Applications for Managing Financial Results* (Armonk, NY: Sharpe Professional, 1997).

5. Arffa, *Expert Financial Planning: Investment Strategies from Industry Leaders* (Wiley, 2011)

6. Hirt, Block, & Basu, [*Investment Planning*](http://www.amazon.com/gp/product/0071437215/ref=as_li_ss_tl?ie=UTF8&tag=maawe-20&linkCode=as2&camp=1789&creative=390957&creativeASIN=0071437215) *for Financial Professionals* (McGraw-Hill, 2006)

7. Janjigian, Horan, & Trzcinka, *The Forbes/CFA Institute Investment Course: Timeless Principles for Building Wealth* ( Wiley, 2011)

8. Shein, *Investment Planning Answer Book* (CCH, 2011)

FREQUENTLY ASKED QUESTIONS

**Question –** Assume you are offered a 10% rate of return on one investment, compounded annually. You are examining alternative investments but they are all compounded on a semiannual or monthly basis. How can you determine which investment will yield the greatest return?

*Answer* – One can restate the question by asking, “what is the effective yield on an investment that has a stated or nominal annual interest rate, but compounds more frequently; most commonly semiannually, quarterly, monthly, or daily?”

Where the number of compounding periods differs from the number of payment periods when comparing alternative investments it is sometimes necessary to convert the stated interest rates to equivalent effective interest rates.

For example, if a $1,000 investment compounds semiannually at an annual stated rate of 10%, the interest earned in the first six months will be added to the original amount of the investment and will thus earn interest for the remainder of the year. Therefore, at the end of 6 months, $50 (10% ∙ $1,000 ∙ ½ year) will be added to the original $1,000 principal. For the balance of the year, $1,050 will be earning interest at the annual rate of 10%. A sum of $52.50 in interest will be earned in the second half of the year (10% ∙ $1,050 ∙ ½ year). The total interest earned for the year on the initial $1,000 therefore is $102.50, resulting in a 10.25% effective rate of return for the year, compounded on a semiannual basis, rather than just the stated rate of 10%.

More frequent compounding results in higher effective interest rates on an investment. For example, a monthly compounding of an investment with a 10% stated annual interest rate has an effective annual rate of 10.471%.

Use the following formula to determine the effective annual interest rate where the nominal annual interest rate is compounded more frequently than annually:



In this formula,

*n* = The Number of Compounding Periods in a Year

*i* = Nominal or Stated Annual Interest Rate

Applying the formula to a nominal annual interest rate of 10% that is compounded quarterly:





Thus, a quarterly compounding of an investment with a 10% stated annual interest rate has an effective annual rate of 10.381%.